

MATHEMATICS

(Three hours)

(Candidates are allowed additional 15 minutes for only reading the paper.

They must **not** start writing during this time)

Section A – Answer **Question 1** (Compulsory) and **five** other questions

Section B and Section C – Answer **two** questions from **either** section B or Section C

All working, including rough work, should be done on the same sheet as and adjacent to, the rest of the answer

The intended marks for questions or parts of questions are given in brackets []

Mathematical tables and graph papers are provided.

Slide rule may be used.

SECTION A

Question 1

- (i) If $\begin{pmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{pmatrix}$ is a symmetric matrix then find the value of x . [3]
- (ii) If $\cos(2\sin^{-1} x) = \frac{1}{9}$ find the values of x [3]
- (iii) Find the value of 'k' if the line $y = 4x + k$ touches the parabola $y^2 = 16x$. [3]
- (iv) Evaluate: $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$ [3]
- (v) Evaluate: $\int \sin \sqrt{x} dx$ [3]
- (vi) Evaluate: $\int_0^{\frac{\pi}{2}} \log \left[\frac{3 + 5 \cos x}{3 + 5 \sin x} \right] dx$ [3]
- (vii) Five books on Mathematics and 3 books on Physics are placed at random on a bookshelf. What is the probability that books on Mathematics are placed side by side on the bookshelf? [3]
- (viii) If the two lines of regression are $4x + y = 225$ and $x + 4y = 150$, find the value of Karl Pearson's coefficient of correlation for the data represented by 'x' and 'y'. [3]
- (ix) If $(\sqrt{3} + i)^6 = x + iy$ find the values 'x' and 'y'. [3]
- (x) Solve $\log \frac{dy}{dx} = 4x - 2y - 2$. [3]
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Question 2

(a) Prove that:

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3 \quad [5]$$

(b) If $A = \begin{pmatrix} -3 & 6 & -1 \\ 2 & 5 & -1 \\ 2 & 4 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 2 & -1 \\ 0 & 5 & -5 \\ -2 & 24 & -27 \end{pmatrix}$ find AB hence or otherwise solve the

following linear equations by matrix method:

$$-3x + 6y - z = 7$$

$$2x + 5y - z = 14$$

$$2x + 4y - z = 11$$

[5]

Question 3

[5]

(a) Verify Lagrange's Mean Value Theorem for the given function:

$$f(x) = x(x+3)(x-2) \text{ on } [-1, 4]$$

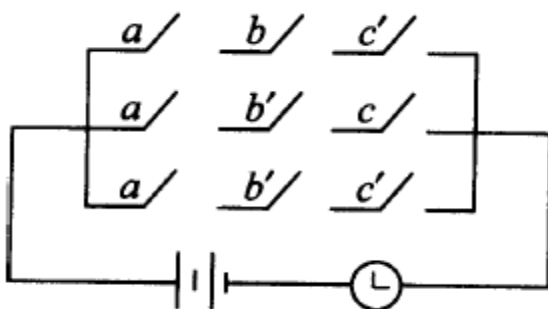
(b) Find the foci and equation of the directrices of the hyperbola

$$9x^2 - 16y^2 + 18x + 64y = 199.$$

Question 4

(a) Prove that $\cos^{-1}\left[\frac{2+3\cos x}{3+2\cos x}\right] = 2\tan^{-1}\left(\frac{1}{\sqrt{5}}\tan\frac{x}{2}\right)$. [5]

(b) (i) Write the Boolean expression corresponding to the switching circuit given below: [5]



(ii) Simplify the expression and construct the switching circuit for the simplified expression.

Question 5

- (a) If $y = \sin(2 \sin^{-1} x)$ show that $(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = 0$. [5]
- (b) Find the dimensions of the rectangle of greatest area that can be inscribed in a semi-circle of radius $10\sqrt{2}$ meters. [5]

Question 6

- (a) Evaluate $\int \frac{x^2 + 1}{x^4 + 1} dx$. [5]
- (b) Find the area of the region bounded between the curves $x = y^2$ and $x = 3 - 2y^2$ [5]

Question 7

- (a) Ten students got the following percentage of marks in Mathematics and Physics:

Marks in Mathematics	56	64	75	85	85	87	91	95	97	98
Marks in Physics	66	72	56	66	74	78	74	88	90	89

Calculate Spearman's coefficient of rank correlation and comment on r. [5]

- (b) For the data given below, find the regression equation of X on Y. Using the equation, calculate the value of X when Y = 15. [5]

X	20	25	30	35	40	45
Y	12	14	16	20	22	25

Question 8

- (a) Two numbers are selected randomly from the first fifteen natural numbers. If the sum of the numbers selected is even, find the probability that both numbers are even. [5]
- (b) In bag A there are 6 white and 4 red balls, while in bag B there are 7 white and 8 red balls and in the bag C, there are 4 white and 3 red balls. One ball is taken out at random from each bag. Find the probability that all the three balls are of the same colour. [5]

Question 9

- (a) Solve : $(1+y^2)dx = (\tan^{-1} y - x)dy$. [5]
- (b) Find the locus of z if $\text{Amp}\left[\frac{z-1}{z+1}\right] = \frac{\pi}{4}$. [5]

SECTION B

Question 10

- (a) With usual notations using vector method prove that in a triangle ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad [5]$$

- (b) Find the volume of the parallelepiped whose three coterminus edges are

$$2i-3j+4k, i+2j-k \text{ and } 3i-j+2k . \quad [5]$$

Question 11

- (a) Determine the equations of the line passing through the point (1,3,-8) and perpendicular to

the lines $\frac{x-2}{1} = \frac{y-5}{2} = \frac{z-1}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+3}{5}$. [5]

- (b) Find the equation of the plane passing through the point $2i - 2j + 2k$ and parallel to the plane $r.(2i - 4j + 2k) = 5$

[5]

Question 12

- (a) The overall percentage of passes in a certain examination is 75. If five candidates from a certain town appear in the examination, what is the probability that at least four pass the examination? [5]

- (b) A consulting firm rents cars from three agencies such that 20% of the cars are rented from agency A, 30% from agency B and 50% from agency C. It is known that 70% of the cars from A, 80% of the cars from B and 90% of the cars from C are in good condition. If a car taken on rent is in good condition, what is probability that it is from agency B? [5]

SECTION C

Question 13

- (a) A furniture firm manufactures chairs and tables, each requiring the use of three machines A, B and C. Production of one chair requires 2 hours on machine A, 1 hour on machine B and 1 hour on machine C. Each table requires 1 hour each on machine A and B and 3 hours on machine C. The profit obtained by selling one chair is Rs. 30 while by selling one table the profit is Rs. 60. The total time available per week on machine A is 70 hours, on machine B is 40 hours and on machine C is 90 hours. How many chairs and tables should be made per week so as to maximize profit? Formulate the problem as L.P.P. and solve it graphically. [5]

- (b) A man borrowed some money and paid it back in three equal quarterly installments of Rs. 9261 each. If the first installment is to be paid one year after the date of borrowing and rate of interest charged was 20% pa compounded quarterly, find the sum he borrowed. Find also total interest charged. [5]

Question 14

- (a) A bill for Rs.56,100 is drawn on 22-1-90 at 11 months and is discounted on 13.10.90 at the rate of 10% . Find the Bankers gain. [5]
- (b) A radio manufacturer finds that he can sell x radios per week at Rs p each, where $p = 2(100 - \frac{x}{4})$. His cost of production of x radios per week is Rs. $(120x + \frac{x^2}{2})$. Find the value of x when marginal cost function and marginal revenue function are equal. [5]

Question 15

- (a) Find the cost of living index number for the year 2005 with 2000 as base from the following data, using weighted average of price relatives: [5]

Commodity	Weights	Price(inRs)	
		2000	2005
Rice	3	15	20
Wheat	3	6	10
Pulses	2	32	40
Milk	1	12	18
Clothing	1	150	200

- (b) Find four quarterly moving average: [5]

Year	Jan-March	April - June	July-Sept.	Oct - Dec.
1980	45	56	39	30
1981	39	49	45	41
1982	46	56	49	40